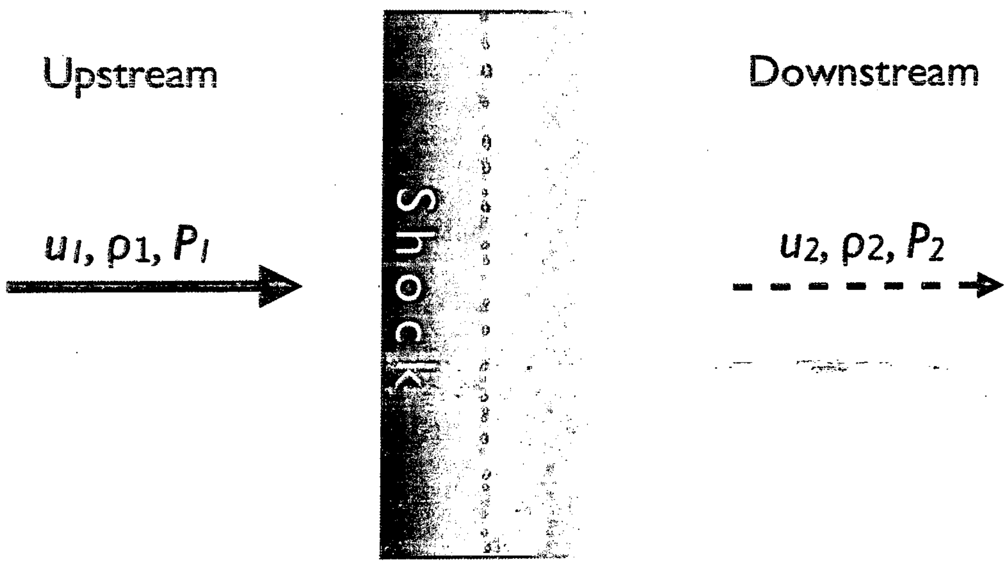
Interstellar shocks

The adiabatic sound speed, $a = (dP/d\rho)^{1/2} = (\gamma P/\rho)^{1/2}$ where P and ρ are the pressure and density of the gas, related by the equation of state, $P \propto \rho^\gamma$. To within 20% (depending on the mean mass of the gas and whether it is molecular, atomic, or ionize), $a \simeq 0.7\,(T/100K)^{1/2}$ km/s. This is relatively small, even for HII regions with $T \simeq 8000\,\mathrm{K}$ compared with typical ISM motions. For such *supersonic* speeds, pressure waves cannot communicate the dynamics of one part of the ISM to another and, instead, there are rapid changes (compression, heating, acceleration) in the physical properties of the gas over small distances, an *interstellar shocks*. Examples include supernova remnants, HII regions, protostellar outflows, and spiral arms.

In general, the shock is very thin, comparable to the mean free path, and we can simply relate the properties of the gas upstream from the shock with those downstream via conservation of mass, momentum and either energy (if radiation is unimportant) or temperature (if we allow for a longer timescale transition region such that the gas cools back down to its preshock value. The picture is as follows *in the frame of the shock*,



The mass flow per unit area per unit time through the shock must be balanced,

$$\rho_1 u_1 = \rho_2 u_2.$$

Similarly the momentum flux per unit area per unit time is $\rho_1 u_1^2$ and must be balanced by the difference in the force per unit area (pressure),

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2.$$

These two (Rankine-Hugioniot) ``jump'' relations can also be directly derived by integrating the static, one-dimensional fluid equations. Note the equations are symmetric even if the physics is not: there is a large, irreversible change of entropy acorss the shoeck.

Assuming radiative losses are small, then we can balance the energy of the gas with the work done by the shock. The energy density is the sum of kinetic and internal terms, $E_1 = \frac{1}{2}\rho_1 u_1^2 + U_1$, where

$$U_1 = \frac{n_f}{2}\frac{N_1 k T_1}{V} = \frac{n_f}{2}\frac{\rho_1 k T_1}{m_1} = \frac{\rho_1 k T_1}{(\gamma-1)m_1} = \frac{P_1}{\gamma-1}$$

and n_f is the number of degrees of freedom, related to the adiabatic index by $\gamma = 1 + 2/n_f$. The energy per unit area per unit time is then $E_1 u_1$. The work done per unit area per unit time is force x distance / (area x time) = pressure x velocity = $P_1 u_1$. Balancing the sum of these acress the shock gives

$$u_1\left[\frac{1}{2}\rho_1 u_1^2 + \frac{P_1}{\gamma-1} + P_1\right] = u_2\left[\frac{1}{2}\rho_2 u_2^2 + \frac{P_2}{\gamma-1} + P_2\right].$$

Note that this assumes γ (the physical state of the gas) does not change across the shock. Now factoring out the density and using the mass conservation equation reduces this to

$$\frac{1}{2}u_1^2 + \frac{\gamma P_1}{(\gamma-1)\rho_1} = \frac{1}{2}u_2^2 + \frac{\gamma P_2}{(\gamma-1)\rho_2}.$$

We see that the second term of the left hand side is related to the sound speed, $a_1^2 = \gamma P_1/\rho_1$. On the right hand side, we substitute $u_2 = \rho_1 u_1/\rho_2$and $P_2 = P_1 + \rho_1 u_1^2 + \rho_2 u_2^2$. After some algebra, we find a quadratic for the density contrast, ρ_2/ρ_1 in terms of γ and the Mach number $M_1 = u_1/a_1$. One solution of the quadratic is the (trivial) solution $\rho_2/\rho_1 = 1$ The other, more interesting solution is

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma+1) + (\gamma-1)(M_1^2 - 1)}.$$

Note that this asymptotes to a finite value for the case of a ``strong shock",

$$\frac{\rho_2}{\rho_1} \to \frac{(\gamma+1)}{(\gamma-1)} \quad \text{as} \quad M_1 \to \infty.$$

For atomic gas with 3 degrees of freedom, $\gamma = 5/3$, and the maximum density contrast is 4. The physical reason for this finte value is energy conservation: without radiative losses, the gas heats up and the thermal pressure resists compression.

With a little more algebra, you can show that the pressure jump,

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1},$$

which relates to the temperature via $P = \rho kT/m$where m is the mean particle mass. For a strong shock with $\gamma = 5/3$,

$$P_1 \rightarrow \frac{3}{4}\rho_1 u_1^2,$$

$$T_1 \rightarrow \frac{3}{16}\frac{mu_1^2}{k}.$$

This conversion of motion to heat is the reason why supernovae produce the hot ionized medium: for $u_1 \sim 10^3$ km/s, $T_1 \sim 10^7$ K.

Now consider the opposite extreme, that of large radiative losses leading to effective cooling and a constant temperature across the shock. For a typical cooling time $\sim 10^4$ years and speed ~ 10 km/s, this occurs over size scales ~ 0.1 pc. On much larger scales, then, we can consider tha gas to be isothermal, $\gamma = 1$, and the density contrast then increases without limit. More formally, we can replace the energy conservation equation above with the jump condition $T_1 = T_2$or equivalently define the isothermal sound speed, $c = (P_1/\rho_1)^{1/2} = (P_2/\rho_2)^{1/2}$. After some algebra, this reduces to the quadratic equation in u_2,

$$u_1 u_2^2 - (c^2 + u_1^2)u_2 + c^2 u_1 = 0,$$

which has the roots $u_2 = u_1$and $u_2 = c^2/u_1 = c/M_1$.The density contrast is then $\rho_2/\rho_1 = u_1/u_2 = M_1^2$ which can be arbitrarily large.

Having now determined the conditions across the shock, we can transform to different frames of reference. In particular, we can look at the relative motion of the gas up and downstream from the shock. Transforming to the upstream frame,

$$u_1 \rightarrow -u_{\rm shock}, \quad u_2 \rightarrow u_2 - u_{\rm shock},$$

implies

$$\frac{u_2}{u_{\rm shock}} = \frac{2}{\gamma+1}\left(1 - \frac{1}{M_1^2}\right).$$

In the case of strong, isothermal shocks, you can see that the gas is accelerated to the shock speed.

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