Spitzer 1978 Physical Processes in the ISM

The condensations produced by such instabilities in a rotating disk have a very large scale, with dimensions comparable to the thickness of the galactic disk; the corresponding masses are roughly 10^6 $M_\odot$. Evidently these considerations are of importance primarily in forming large aggregations, or cloud complexes. As we have seen [S11.3b], a magnetic field of order 3×10^{-6} G will not hinder the gravitational contraction of such large masses. However, the concentration of gas in magnetic valleys or troughs [S11.2c], near the galactic midplane, may be an equally effective method for forming such large aggregations. Since we know from observations that such large concentrations of gas exist within spiral arms, the method by which these cloud complexes form is not a necessary part of star formation theory.

b. Gravitational Collapse of a Sphere

When a cloud departs from equilibrium, either because of gravitational instability or because its mass or external pressure exceeds the limits for possible equilibria, contraction will generally follow. If the temperature is assumed constant, the contraction of a spherical cloud of radius R will be an accelerating process. The gravitational force per cm^3 varies as ρ/R^2 or as M/R^5, whereas the force due to the pressure gradient varies as $p(0)/R$ or $MT(0)/R^4$, where $p(0)$ and $T(0)$ are the pressure and temperature at the center. Thus if the contraction is isothermal, pressure forces cannot retard the collapse. Magnetic forces also are unimportant if the mass exceeds a critical value [S11.3b], although as we shall see below angular momentum can stop the contraction. If the temperature rises adiabatically, with $\gamma>4/3$, pressure forces can also arrest the contraction, but as long as the cloud has a sufficiently low opacity to remain isothermal, a cloud that starts contracting spherically as a result of its self-gravitational attraction will contract with an increasing acceleration if its angular momentum is sufficiently low.

This type of accelerating isothermal collapse is not possible for contraction in one or even two dimensions. For the contraction of a plane parallel sheet, for example, the acceleration g perpendicular to the sheet for any layer is unaffected by changes in the sheet thickness, whereas the corresponding pressure force per gram, $\nabla p/\rho$, tends to increase as the sheet is compressed, reducing the acceleration as the contraction proceeds. The two-dimensional case is intermediate in that the gravitational and pressure forces change in the same way with cylindrical radius r if $T(0)$ remains constant.

We compute first the collapse time for a cold sphere of uniform density $\rho(t)$ on the assumption that initially, when $t=0$, the sphere is at rest. We let $r(t)$ be the radius of a particular mass shell as a function of time. The value of $r(0)$ will be denoted by a, and we omit the argument t from ρ and r. The equation of motion then becomes

$$\frac{d^2r}{dt^2}=-\frac{GM(a)}{r^2}=-\frac{4\pi G\rho(0)a^3}{3r^2}, \tag{13-44}$$

where $M(a)$ is the mass interior to the initial radius of the shell; evidently the mass inside the shell stays constant during the collapse, if the shells are assumed not to cross each other. Multiplying equation (13-44) by dr/dt and integrating gives the energy integral

$$\frac{dr}{adt}=-\left[\frac{8\pi G\rho(0)}{3}\left(\frac{a}{r}-1\right)\right]^{1/2}. \tag{13-45}$$

If we make the substitution $r/a=\cos^2\beta$, equation (13-45) yields [20]

$$\beta+\tfrac{1}{2}\sin 2\beta=t\left[\frac{8\pi G\rho(0)}{3}\right]^{1/2}, \tag{13-46}$$

where we let t vanish initially, when dr/dt is zero. Evidently β is the same for all mass shells at any one time, and all shells reach the center at the same time, when β equals $\pi/2$, and after a "free-fall time" given by

$$t_f=\left\{\frac{3\pi}{32G\rho(0)}\right\}^{1/2}=\frac{4.3\times10^7}{\{n_{\rm H}(0)\}^{1/2}}\text{ years.} \tag{13-47}$$

We consider next the situation where the initial density is a function of initial radius a. In this case we denote by $\rho_m(a,t)$ the mean density at the time t within the mass shell of initial radius a. We assume again that different shells do not cross each other; hence $M(a)$ is again constant with time. Equations (13-44) through (13-47) then are valid as before, provided that $\rho_m(a,0)$ replaces $\rho(0)$ throughout. The free-fall time is now different for different mass shells, and the condition that different shells do not cross will be satisfied provided that $\rho_m(a,0)$ decreases with increasing a; that is, that the initial density decreases outward. The inner regions will then collapse first, with the outer ones falling in at progressively later times. We assume that the shells do not "bounce"; that is, that all mass shells reaching the center stay there. Under these conditions it is readily seen that the density distribution must become very peaked toward the